$\sim$

7°

$\sim 10'$

$$N_{\mathrm{pix}} \sim 10^7$$

$$\cos\theta = a \pm b \cdot \phi$$

$$\cos\theta = a + b/\phi^2$$

$$\cos\theta = a + b/(\pi/2 - \phi)^2$$

$$2^{\mathrm{floor}(\log_2(u/4)/2)},$$

$$4p,\ 4p+1,\ 4p+2,\ 4p+3$$

$$4u,\ 4u+1,\ 4u+2,\ 4u+3$$

$$Y_{\ell m}$$

$f(\gamma)$

=

$$\sum_{\ell=0}^{\ell_{\max}} \sum_{m} a_{\ell m} Y_{\ell m}(\gamma),$$

γ

$\theta \in [0, \pi]$

$\phi \in [0, 2\pi)$

$$\ell > \ell_{\max}$$

$-\ell_{\max}$

$\ell_{\max}$

ℓm

$$m > \ell$$

$$f(\gamma)$$

γ_p

$$p \in [0, N_{\text{pix}} - 1]$$

$a_{\ell m}$

$$\hat{a}_{\ell m}$$

$$\frac{4\pi}{N_{\text{pix}}} \sum_{p=0}^{N_{\text{pix}}-1} Y^*_{\ell m}(\gamma_p) f(\gamma_p),$$

$\hat{a}_{\ell m}$

$$\hat{C}_\ell$$

$$\hat{C}_\ell$$

$$\frac{1}{2\ell+1}\sum_m |\hat{a}_{\ell m}|^2.$$

$$f(\gamma_p) \longrightarrow f(\gamma_p)w(\gamma_p)$$

C_ℓ

$$f(\gamma_p)$$

ℓ

$$f(\gamma)/\sqrt{4\pi}$$

$$1/(4\pi)$$

$f(\gamma)$

4π

$$(2\ell + 1)C_\ell$$

$$2\ell + 1$$

$\Delta T/T$

$$\Delta T$$

μK

$$C_{X,\ell}$$

$D_{X,\ell}$

$$\frac{\ell(\ell+1)}{(2\pi)T_{CMB}^2}C_{X,\ell},$$

$$C_X(\ell)$$

$$\frac{\ell(\ell+1)}{2\pi} C_X(\ell)$$

μ

$$a_{21}^{TEMP} = - a_{2-1}^{TEMP} = 1$$

$$a_{21}^{GRAD} = -\, a_{2-1}^{GRAD} = 1$$

$$2 \times 2$$

ψ

$$\mathbf{e}_1{}' = \cos\psi\ \mathbf{e}_1 + \sin\psi\ \mathbf{e}_2$$

$$\mathbf{e}_2{}' = -\sin\psi\ \mathbf{e}_1 + \cos\psi\ \mathbf{e}_2$$

Q'

$$\cos 2\psi\ Q + \sin 2\psi\ U$$

$$U'$$

$$-\sin 2\psi\ Q + \cos 2\psi\ U$$

$$Q \pm iU$$

± 2

$$_{\pm 2}Y_l^m$$

$$\sum_{lm} a_{T,lm} Y_{lm}(\mathbf{n})$$

$$\sum_{lm} a_{2,lm}\ {}_2Y_{lm}(\mathbf{n})$$

$$\sum_{lm} a_{-2,lm} \; {}_{-2}Y_{lm}(\mathbf{n}).$$

$$(\mathbf{e}_1, \mathbf{e}_2) = (\mathbf{e}_\theta, \mathbf{e}_\phi)$$

$\mathbf{e}_\theta$

$\mathbf{e}_{\phi}$

$$\pm_2 a_{lm}$$

$$\langle a^*_{X,lm} a_{X,lm'} \rangle$$

$$\delta_{m,m'} C_{Xl} \quad \langle a^*_{T,lm} a_{E,lm} \rangle = \delta_{m,m'} C_{Cl},$$

$$\left\langle \cdot\ \cdot\ \cdot \right\rangle$$

$$\delta_{i,j}$$

$$-\sum_{lm} a_{E,lm} X_{1,lm} + i a_{B,lm} X_{2,lm}$$

$$-\sum_{lm} a_{B,lm} X_{1,lm} - i a_{E,lm} X_{2,lm}$$

$$X_{1,lm}(\mathbf{n}) = ({}_2Y_{lm} + {}_{-2}Y_{lm})/2$$

$$X_{2,lm}(\mathbf{n}) = (\ {}_2Y_{lm} - \ {}_{-2}Y_{lm})/2$$

$$X_{1,lm}(\mathbf{n}) = \sqrt{(2l+1)/4\pi} F_{1,lm}(\theta)\ e^{im\phi}$$

$$X_{2,lm}(\mathbf{n}) = \sqrt{(2l+1)/4\pi}F_{2,lm}(\theta)\ e^{im\phi}$$

$$F_{(1,2),lm}(\theta)$$

$F_{1,lm}(\theta)$

$$N_{lm}\left[-\left(\frac{l-m^2}{\sin^2\theta}+\frac{1}{2}l(l-1)\right)P_l^m(\cos\theta)+(l+m)\frac{\cos\theta}{\sin^2\theta}P_{l-1}^m(\cos\theta)\right]$$

$$F_{2,lm}(\theta)$$

$$N_{lm}\frac{m}{\sin^2\theta}[-(l-1)\cos\theta P_l^m(\cos\theta)+(l+m)P_{l-1}^m(\cos\theta)],$$

$$N_{lm}(\theta)$$

$$2\sqrt{\frac{(l-2)!(l-m)!}{(l+2)!(l+m)!}}.$$

$$F_{2,lm}(\theta) = 0$$

β

$$\sum_{m} {}_{s_1}Y^*_{lm}(\mathbf{n}_1)\ {}_{s_2}Y_{lm}(\mathbf{n}_2)$$

$$\sqrt{\frac{2l+1}{4\pi}}\; {}_{s_2}Y_{l-s_1}(\beta, \psi_1) e^{-is_2\psi_2}$$

$$\langle T_1 T_2 \rangle$$

$$\sum_l \frac{2l+1}{4\pi} C_{Tl} P_l(\cos\beta)$$

$$\langle Q_r(1) Q_r(2) \rangle$$

$$\sum_l \frac{2l+1}{4\pi}[C_{El}F_{1,l2}(\beta) - C_{Bl}F_{2,l2}(\beta)]$$

$$\langle U_r(1) U_r(2) \rangle$$

$$\sum_l \frac{2l+1}{4\pi}[C_{Bl}F_{1,l2}(\beta) - C_{El}F_{2,l2}(\beta)]$$

$$\langle T(1)Q_r(2)\rangle$$

$$-\sum_{l} \frac{2l+1}{4\pi} C_{Cl} F_{1,l0}(\beta)$$

$$\langle T(1)U_r(2)\rangle$$

$$\beta \to 0$$

$$P_\ell(\cos\beta) \to 1$$

$$P_\ell^2(\cos\beta) \to \sin^2\beta\,\frac{(\ell+2)!}{8(\ell-2)!}$$

$$\langle TT \rangle$$

$$\sum_{\ell} \frac{2\ell + 1}{4\pi} C_{T\ell}$$

$$\langle QQ \rangle + \langle UU \rangle$$

$$\sum_{l} \frac{2\ell + 1}{4\pi} \left(C_{E\ell} + C_{B\ell}\right)$$

$$\langle TQ \rangle = \langle TU \rangle$$

$$C_{C\ell}$$

$$\begin{pmatrix} Q' \\ U' \end{pmatrix} = \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ -\sin 2\psi & \cos 2\psi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix},$$

$$\begin{pmatrix} a'_{E,\ell m} \\ a'_{B,\ell m} \end{pmatrix} = \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ -\sin 2\psi & \cos 2\psi \end{pmatrix} \begin{pmatrix} a_{E,\ell m} \\ a_{B,\ell m} \end{pmatrix}.$$

$\geq$

$\leq$

$$C_\ell^{\mathrm{TEMP}}$$

$C_{\mathrm{T},\ell}$

$$C_\ell^{\mathrm{T}}$$

$$C_\ell^{\mathrm{GRAD}}$$

$C_{\mathrm{E},\ell}$

$$2C_\ell^{\mathrm{G}}$$

$$2C_\ell^{\mathrm{GRAD}}$$

$$C_\ell^{\mathrm{CURL}}$$

$$C_{\mathrm{B},\ell}$$

$$2C_\ell^{\mathrm{C}}$$

$$2C_\ell^{\mathrm{CURL}}$$

$$C_\ell^{\mathrm{T-GRAD}}$$

$$C_{\mathrm{C},\ell}$$

$$-\sqrt{2}$$

$$C_\ell^{\mathrm{TG}}$$

$$\sqrt{2}C_{\ell}^{\mathrm{T-GRAD}}$$

$M_{\ell m}$

$$\begin{pmatrix} X_{1,\ell m} & iX_{2,\ell m} \\ -iX_{2,\ell m} & X_{1,\ell m} \end{pmatrix}$$

$$\begin{pmatrix} Q \\ U \end{pmatrix}$$

$$\sum_{\ell m} M_{\ell m} \begin{pmatrix} -a_{\ell m}^{\mathrm{GRAD}} \\ -a_{\ell m}^{\mathrm{CURL}} \end{pmatrix}.$$

$$\begin{pmatrix} Q \\ -U \end{pmatrix}$$

$$\sum_{\ell m} M_{\ell m} \begin{pmatrix} \sqrt{2} a_{\mathrm{E},\ell m} \\ \sqrt{2} a_{\mathrm{B},\ell m} \end{pmatrix},$$

$$\begin{pmatrix} Q \\ U \end{pmatrix}$$

$$\sum_{\ell m} M_{\ell m} \begin{pmatrix} -\sqrt{2} a_{\ell m}^{\mathrm{GRAD}} \\ \sqrt{2} a_{\ell m}^{\mathrm{CURL}} \end{pmatrix}.$$

$$Y_{\ell m}(\theta, \phi)$$

$$\lambda_{\ell m}(\cos\theta)e^{im\phi}$$

$$\lambda_{\ell m}(x)$$

$$\sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}P_{\ell m}(x), \quad \text{for } m \geq 0$$

$\lambda_{\ell m}$

$$(-1)^m \lambda_{\ell|m|}, \quad \text{for } m < 0,$$

$$0, \quad \text{for } |m| > \ell.$$

$$x \equiv \cos\theta$$

$$P_{\ell m}$$

$$(1-x^2)\frac{d^2}{dx^2}P_{\ell m} - 2x\frac{d}{dx}P_{\ell m} + \left(\ell(\ell+1) - \frac{m^2}{1-x^2}\right)P_{\ell m}$$

P_ℓ

$P_{\ell m}$

$$(-1)^m(1-x^2)^{m/2}\frac{d^m}{dx^m}P_\ell(x),$$

$P_\ell(x)$

$$\frac{1}{2^\ell \ell!} \frac{d^\ell}{dx^\ell}(x^2 - 1)^\ell.$$

$$\Omega_{\text{pix}}$$

$$\int d\mathbf{u} w_p(\mathbf{u}) f(\mathbf{u})$$

$$1/\Omega_{\rm pix}$$

$$\int d\mathbf{u} w_p(\mathbf{u}) = 1$$

$$\sum_{\ell=0}^{\ell_{\max}} \sum_{m} a_{\ell m} w_{\ell m}(p),$$

$$w_{\ell m}(p)$$

$$\int d\mathbf{u} w_p(\mathbf{u}) Y_{\ell m}(\mathbf{u}),$$

$$w_{\ell m}(p)$$

$$w_\ell(p) Y_{\ell m}(p)$$

$$w_\ell(p)$$

$$\left(\frac{4\pi}{2\ell + 1} \sum_{m=-\ell}^{\ell} |w_{\ell m}(p)|^2 \right)^{1/2},$$

$$C_\ell^{\text{pix}}$$

$$C_\ell^{\text{unpix}}$$

$$C_{\ell}^{\mathrm{pix}}$$

$$w_\ell^2 C_\ell^{\mathrm{unpix}}$$

w_ℓ

w_ℓ

$$\left(\frac{1}{N_{\mathrm{pix}}} \sum_{p=0}^{N_{\mathrm{pix}}-1} w_\ell^2(p) \right)^{1/2} .$$

$$\ell \leq 4N_{\text{side}}$$

$N_{\rm side} \leq 128$

$$\Delta w/w < 7\ 10^{-4}$$

$$\ell = 2N_{\rm side}$$

$$\Delta w/w < 1.7\ 10^{-3}$$

$$\ell = 4N_{\mathrm{side}}$$

$x \in]0, 1[$

$$2^{128} - 1 \approx 3.410^{38}$$